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Noncommutative Compactifications of Type I Strings on Tori with Magnetic Background Flux

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Abstract

We construct six- and four-dimensional toroidal compactifications of the Type I string with magnetic flux on the D-branes. The open strings in this background probe a noncommutative internal geometry. Phenomenologically appealing features such as chiral fermions and supersymmetry breaking in the gauge sector are naturally realized by these vacua. We investigate the spectra of such noncommutative string compactifications and in a bottom-up approach discuss the possibility to obtain the standard or some GUT like model.

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1. Introduction

The search for realistic string vacua is one of the burning open problems within superstring theory. A phenomenologically viable string compactification should contain at least three chiral fermion generations, the standard model gauge group and broken space-time supersymmetry. In the context of 'conventional' string compactifications the requirement of getting chiral fermions is usually achieved by considering compact, internal background spaces with nontrivial topology rather than simple tori. In particular, when analyzing the Kaluza-Klein fermion spectra [1] a net-fermion generation number arises if the internal Dirac operator has zero modes. For example, considering heterotic string compactifications on Calabi-Yau threefolds [2], the net-generation number is equal to $|\chi|/2$, where χ is the Euler number of the Calabi-Yau space. Chiral fermions are also present in a large class of heterotic orbifold compactifications [3], as well as in free bosonic [4] and fermionic [5] constructions. Type II string models with chiral fermions can be constructed by locating D-branes at transversal orbifold or conifold singularities [6], or by considering intersections of D-branes and NS-branes [7]; chiral type I models were first proposed in [8]. Moreover orbifold compactificactions of eleven-dimensional M-theory can lead to chiral fermions, as discussed e.g. in [9].

The phenomenological requirement of breaking space-time supersymmetry can be met in various ways. In the context of heterotic string compactifications gaugino condensation [10] or the Scherk-Schwarz mechanism [11,12] lead to potentially interesting models with supersymmetry broken at low energies. In addition, as it was realized more recently, type II models on nontrivial background spaces with certain D-brane configurations possess broken space-time supersymmetry. Especially, when changing the GSO-projections tachyon free type 0 orientifolds in four dimensions can be constructed [13]. Alternatively, orientifolds on six-dimensional orbifolds with brane-antibrane configurations provide interesting scenarios [14], where supersymmetry is left unbroken in the gravity bulk, but broken in the open string sector living on the brane-antibrane system.

Finally the quest for a realistic gauge group with sufficiently low rank is met in heterotic strings by choosing appropriate gauge vector bundles on the Calabi-Yau spaces [15], which can be alternatively described by turning on Wilson lines in Calabi-Yau or also in orbifold compactifications [16]. On the type II side the rank of the gauge group can be also lowered by Wilson lines or, in the T-dual picture, by placing the branes at different positions inside the internal space.

As it should have become clear from the previous discussion, ‘standard’ heterotic, type I or type II compactifications on simple 6-tori do not meet any of the three above requirements. However, as we will discuss in this paper, turning on magnetic fluxes in the internal directions of the D-branes, thereby inducing mixed Neumann-Dirichlet boundary conditions for open strings equivalent to a noncommutative internal geometry [17] on the branes, all three goals can be achieved in one single stroke.[1] Specifically, we will discuss type I string compactifications on a product of d noncommutative two-tori to $10-2d$ non-compact Minkowski dimensions ($d=2,3$), i.e. the ten-dimensional background spaces M^{10} we are considering have the following form

$$M^{10} = R^{1,9-2d} \times \prod_{j=1}^{d} T^2_{(j)}. \tag{1.1}$$

The coordinates in the internal space possess the following commutation relations

$$[X_{10-2j}, X_{11-2j}] = i\theta^{(j)}\,, \qquad j=1,\ldots,d. \tag{1.2}$$

These commutation relations will be realized by D9-branes with constant background magnetic fluxes $F^{(j)}$ turned on in the directions of the 2-tori [19-22], corresponding to the following noncommutative deformation parameter $\theta^{(j)}$ in eq.(1.2):

$$\theta^{(j)} = -\frac{2\pi\alpha' F^{(j)}}{1+(F^{(j)})^2}. \tag{1.3}$$

The entire internal noncommutative torus will actually consist out of different sectors with different noncommutative deformation parameters, because we will introduce several D9-branes with different magnetic fluxes. We will show that the spectrum of open strings, with mixed boundary conditions in the internal directions is generically chiral, breaks space-time supersymmetry and leads to gauge groups of lower rank. It is however important to stress that the effective gauge theories in the uncompactified part of space-time are still commutative, and therefore are Lorentz invariant and local field theories.

This construction is the D-brane extended version of [23], where it was already observed that turning on magnetic flux in a toroidal type I compactification leads to supersymmetry breaking and chiral massless spectra in four space-time dimensions. However,

[1] In a previous paper [18] we have discussed type I string compactifications on noncommutative asymmetric orbifold spaces.

the consistency conditions for such models were derived in the effective non-supersymmetric gauge theories, leaving the actual string theoretic conditions an open issue. We will show that, with all the insights gained in the description of D-branes with magnetic flux, we are now able to achieve a complete string theoretic understanding, giving rise to certain extensions and modifications of the purely field theoretical analysis. As a solution to the tadpole cancellation conditions we can get different sectors of D-branes with different magnetic fluxes, corresponding to different noncommutative boundary conditions. Chirality then arises in sectors of open strings which have ends on branes with different gauge flux, while the presence of any solitary flux is not sufficient. The gauge groups that act on the D-branes with non-vanishing flux are unitary instead of orthogonal or symplectic in accord with the general statement that only these are compatible with a noncommutative deformation of the coordinate algebra.

For this kind of models, it is sometimes very helpful to employ an equivalent T-dual description, where the background fields vanish and the torus is entirely commutative, but the D-branes intersect at various different angles [24]. This description allows to present a more intuitive picture of the open string sector involved in such models. Chiral fermions then arise due to the nontrivial geometric boundary conditions of the intersecting D-branes, which at the same time also break space-time supersymmetry and lower the rank of the gauge group.

The paper is organized as follows. In the next section we analyze the one-loop amplitudes and the resulting tadpole cancellation conditions for D9-branes with mixed Neumann-Dirichlet boundary conditions moving in the background of d two-dimensional tori ($d = 2, 3$). In section 3 we discuss specific six-dimensional models ($d = 2$) working out the non-supersymmetric, chiral spectrum. We also point out some subtleties involving the mechanisms of supersymmetry breaking in 'nearly' supersymmetric brane configurations. In section 4 we move on to chiral, non-supersymmetric four-dimensional models ($d = 3$), reconsider in particular the model presented in [23] with GUT-like gauge group $G = U(5) \times U(3) \times U(4) \times U(4)$ and display another 4 generation model with 'standard model' gauge group $G = U(3) \times U(2) \times U(1)^r$.[2] Some phenomenological problems of this model are stressed at the end.

[2] For other recent bottom up attempts to obtain GUT's and the standard model from branes see [25].

2. One loop amplitudes

In [23] it was observed that turning on magnetic flux in a toroidal type I compactification leads to supersymmetry breaking and in general to chiral massless spectra in four space-time dimensions. The consistency conditions for such models were derived in the effective non-supersymmetric gauge theory but not in the full string theory. In this section we will show that, with the inclusion of D-branes with magnetic flux, respectively D-branes at angles, we are now able to derive the string theoretic tadpole cancellation conditions.

2.1. D9-branes with magnetic fluxes

As our starting point we consider the orientifold

$$\frac{\text{Type IIB on } T^{2d}}{\Omega}, \tag{2.1}$$

where Ω denotes the word-sheet parity transformation. In the following we will assume that T^{2d} splits into a direct product of d two-dimensional tori $T^2_{(j)}$ with coordinates $X_1^{(j)}$, $X_2^{(j)}$ and radii $R_1^{(j)}$, $R_2^{(j)}$, $j = 1, \ldots, d$. Their complex structures will always be taken to be purely imaginary, and the antisymmetric NSNS tensorfield will be set to zero. Turning on magnetic flux, $F_{12}^{(j)} = F^{(j)}$, on a D9-brane changes the pure Neumann boundary conditions into mixed Neuman-Dirichlet conditions

$$\begin{aligned} \partial_\sigma X_1^{(j)} + F^{(j)} \partial_\tau X_2^{(j)} &= 0, \\ \partial_\sigma X_2^{(j)} - F^{(j)} \partial_\tau X_1^{(j)} &= 0, \end{aligned} \tag{2.2}$$

Let us consider different kinds of D9_μ-branes, labelled by $\mu \in \{1, \ldots, K\}$, distinguished by different magnetic fluxes on at least one torus. Specifically we are only considering branes which are characterized by d sets of two integers $(n_\mu^{(j)}, m_\mu^{(j)})$ corresponding to the electric respectively magnetic charges[3].

The magnetic fluxes of such a brane are given by

$$F_\mu^{(j)} = \frac{m_\mu^{(j)}}{n_\mu^{(j)} R_1^{(j)} R_2^{(j)}}. \tag{2.3}$$

[3] All models considered in [23] correspond to the subset of branes with $n_\mu^{(j)} = 1$ for all j and μ.

Following [21] the boundary conditions eq.(2.2) define an open string metric on $T^2_{(j)}$ in the following way:

$$G^{(j)kl}_\mu = \frac{1}{1+(F^{(j)}_\mu)^2}\,\delta^{kl}. \tag{2.4}$$

The deformation parameter of the noncommutative torus is given by

$$\theta^{(j)kl}_\mu = -\frac{2\pi\alpha' F^{(j)}_\mu}{1+(F^{(j)}_\mu)^2}\,\epsilon^{kl}. \tag{2.5}$$

Let us now discuss the Kalazu-Klein mass spectrum of open strings on the noncommutative torus. Since the translations on the torus are not deformed by eq.(1.2), the center of mass Kaluza-Klein momenta on $T^2_{(j)}$ are unchanged and are given by

$$p^{(j)}_{\mu 1} = \frac{r^{(j)}_\mu}{n^{(j)}_\mu R^{(j)}_1}, \qquad p^{(j)}_{\mu 2} = \frac{s^{(j)}_\mu}{n^{(j)}_\mu R^{(j)}_2}. \tag{2.6}$$

Here the electric charge of the brane enters as the number of times it wraps the torus while the integers $r^{(j)}_\mu$ and $s^{(j)}_\mu$ are the two Kaluza-Klein momentum numbers along $T^2_{(j)}$. Howover the masses of the open string Kalazu-Klein states do depend on the magnetic fluxes. Using the boundary state formalism one can compute the mass spectrum of the open string Kalazu-Klein states on $T^2_{(j)}$ with mixed boundary conditions specified by the two integers $n^{(j)}_\mu$ and $m^{(j)}_\mu$ with the following result [22],[18]:

$$(M^{(j)}_\mu)^2 = \frac{(s^{(j)}_\mu R^{(j)}_1)^2 + (r^{(j)}_\mu R^{(j)}_2)^2}{(m^{(j)}_\mu)^2 + (n^{(j)}_\mu R^{(j)}_1 R^{(j)}_2)^2}. \tag{2.7}$$

This allows use to extract the 'noncommutative' Kaluza-Klein momenta and winding contributions

$$\begin{aligned}
\tilde p^{(j)}_\mu &= \frac{n^{(j)}_\mu R^{(j)}_1 R^{(j)}_2}{(m^{(j)}_\mu)^2 + (n^{(j)}_\mu R^{(j)}_1 R^{(j)}_2)^2}\left(r^{(j)}_\mu R^{(j)}_2 + i s^{(j)}_\mu R^{(j)}_1\right) = \frac{1}{1+(F^{(j)}_\mu)^2}\left(p^{(j)}_{\mu 1} + i p^{(j)}_{\mu 2}\right),\\
w^{(j)}_\mu &= \frac{m^{(j)}_\mu}{(m^{(j)}_\mu)^2 + (n^{(j)}_\mu R^{(j)}_1 R^{(j)}_2)^2}\left(-s^{(j)}_\mu R^{(j)}_1 + i r^{(j)}_\mu R^{(j)}_2\right) = \frac{F^{(j)}_\mu}{1+(F^{(j)}_\mu)^2}\left(-p^{(j)}_{\mu 2} + i p^{(j)}_{\mu 1}\right),
\end{aligned} \tag{2.8}$$

(here written in complex notation) where the mass formula eq.(2.7) is given as $(M^{(j)}_\mu)^2 = |\tilde p^{(j)}_\mu|^2 + |w^{(j)}_\mu|^2$. Of course, for $F^{(j)}_\mu = 0$ the momenta $p^{(j)}_\mu$ and $\tilde p^{(j)}_\mu$ agree, and $w^{(j)}_\mu = 0$. Note that the KK masses in eq.(2.7) can be also expressed by using the standard KK momenta eq.(2.6) togother with the open string metric eq.(2.4) as

$$(M^{(j)}_\mu)^2 = p^{(j)}_{\mu k} G^{(j)kl}_\mu p^{(j)}_{\mu l}. \tag{2.9}$$

To convince ourselves that we are indeed dealing with a noncommutative torus for open strings which end on the $D9_\mu$-brane with magnetic flux $F_\mu^{(j)}$, let us compute the OPE of the vertex operators $\mathcal{O}(z) = e^{ip_\mu^{(j)} X^{(j)}}(z)$ [20],[21]:

$$
\begin{aligned}
e^{ip_\mu^{(j)} X^{(j)}}(\tau)\, e^{iq_\mu^{(j)} X^{(j)}}(\tau') = (\tau - \tau')^{\frac{2\alpha' p_\mu^{(j)} q_\mu^{(j)}}{1+(F_\mu^{(j)})^2}} \\
\exp\left(-i\pi\alpha' \frac{F_\mu^{(j)}}{1+(F_\mu^{(j)})^2} \epsilon_{kl} p_{\mu k}^{(j)} q_{\mu l}^{(j)}\right) e^{i(p_\mu^{(j)}+q_\mu^{(j)})X^{(j)}}(\tau') + \ldots.
\end{aligned}
\tag{2.10}
$$

For generic, internal momenta $p_\mu^{(j)}$, $q_\mu^{(j)}$ in eq.(2.6), the phase factor is indeed nontrivial, and hence the torus is noncommutative. The commutator of the coordinates of the open string endpoints on the $D9_\mu$-brane is given by [26],[27]

$$
[X_1^{(j)}(\tau), X_2^{(j)}(\tau)] = -\frac{2\pi i\alpha' F_\mu^{(j)}}{1+(F_\mu^{(j)})^2}\,, \tag{2.11}
$$

in agreement with eqs.(1.2),(2.5).

2.2. $D(9-d)$-branes at angles

Instead of working with D9-branes with various magnetic fluxes, we will now use the T-dual description in terms of D-branes at angles [24], which allows to present a more intuitive picture of the open string sector involved in such models. Applying a T-duality in all $X_2^{(j)}$ directions

$$
R_2^{(j)} \to R_2^{(j)'} = 1/R_2^{(j)}, \tag{2.12}
$$

leads to boundary conditions for D$(9-d)$-branes intersecting at angles, where the angle of the D$(9-d)$-brane relative to the $X_1^{(j)}$ axes is given by

$$
\tan\phi^{(j)} = F^{(j)}. \tag{2.13}
$$

(In the following we will omit the prime on the dual radii.) This T-duality also maps Ω onto $\Omega\mathcal{R}$, where $\mathcal{R}$ acts as complex conjugation on all the d complex coordinates along the $T^2_{(j)}$ tori. Thus, instead of (2.1) we are considering the orientifold

$$
\frac{\text{Type II on } T^{2d}}{\Omega\mathcal{R}}. \tag{2.14}
$$

For d even we have to take type IIB and for d odd type IIA. Note that after performing this T-duality transformation the internal coordinates are completely commutative.

Let $j \in \{1, \ldots, d\}$ again label the d different two-dimensional tori and $\mu \in \{1, \ldots, K\}$ the different kinds of D$(9-d)$-branes, which are distinguished by different angles on at least one torus. Moreover, we are only considering branes which do not densely cover any of the two-dimensional tori. Thus, the position of a D$(9-d)$-brane is described by two sets of integers $(n_\mu^{(j)}, m_\mu^{(j)})$, labelling how often the D-branes are wound around the two fundamental cycles of each $T^2_{(j)}$. The angles of such a brane with the axes $X_1^{(j)}$ are given by

$$\tan \phi_\mu^{(j)} = \frac{m_\mu^{(j)} R_2^{(j)}}{n_\mu^{(j)} R_1^{(j)}}. \tag{2.15}$$

All these conventions are shown in figure 1.

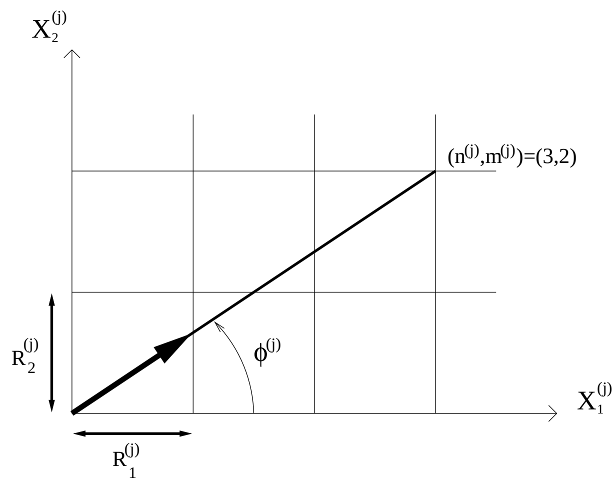

Figure 1

Since $\Omega\mathcal{R}$ reflects the D-branes at the axis $X_1^{(j)}$, for each brane labelled by $(n_\mu^{(j)}, m_\mu^{(j)})$ we must also introduce the mirror brane with $(n_{\mu'}^{(j)}, m_{\mu'}^{(j)}) = (n_\mu^{(j)}, -m_\mu^{(j)})$. The values $m_\mu^{(j)} = 0 \neq n_\mu^{(j)}$ and $n_\mu^{(j)} = 0 \neq m_\mu^{(j)}$ correspond to branes located along one of the axis. The horizontal D-branes translate via T-duality into D9-branes with vanishing flux and the vertical ones into branes of lower dimension with pure Dirichlet boundary conditions.

The questions we are going to deal with in the following are: Is it possible to cancel all or at least the RR tadpoles originating from the Klein bottle amplitude by D9-branes with non-vanishing magnetic fluxes $F^{(j)}$, or equivalently by D$(9-d)$-branes at nontrivial angles $\phi^{(j)}$? Taken that supersymmetry is broken generically by such a background, are there configuration which still preserve some amount of supersymmetry? This would provide a string scenario with partial supersymmetry breaking. Finally, what are the phenomenological properties of such compactifications? Concerning the first question we find

a positive answer in the sense that the RR tadpole can be cancelled, while supersymmetry is always broken entirely, albeit sometimes in a rather subtle manner. This comes along with a non-vanishing NSNS tadpole and the presence of tachyons. Interestingly, these models generically contain chiral fermions motivating us to study how far one can get in deriving the standard model in this setting. However, later we will mention an obstacle to construct phenomenologically realistic models in this simple approach.

Technically we first have to compute all contributions to the massless RR tadpole. The cancellation conditions will then imply relations for the number of D9-branes and their respective background fluxes. This computation will be performed in the T-dual picture, where D9-branes with background fields are mapped to D$(9-d)$-branes, and the background fields translate into relative angles. This picture allows to visualize the D-branes easily and gives a much better intuition than dealing with sets of D9-branes, all filling the same space but differing by background fields.

2.3. Klein bottle amplitude

The loop channel Klein bottle amplitude for (2.14) can be computed straightforwardly

$$\mathcal{K} = 2^{(5-d)} c\,(1-1) \int_0^\infty \frac{dt}{t^{(6-d)}} \frac{1}{4} \frac{\vartheta\left[{0 \atop 1/2}\right]^4}{\eta^{12}} \prod_{j=1}^{d} \left[\sum_{r,s\in\mathbb{Z}} e^{-\pi t \left(r^2/\left(R_1^{(j)}\right)^2 + s^2 \left(R_2^{(j)}\right)^2 \right)} \right], \qquad (2.16)$$

with $c = V_{10-2d}/\left(8\pi^2\alpha'\right)^{5-d}$. Transforming (2.16) into tree channel, one obtains the following massless RR tadpole

$$\int_0^\infty dl\, 2^{(13-d)} \prod_{j=1}^{d} \left(\frac{R_1^{(j)}}{R_2^{(j)}} \right). \qquad (2.17)$$

The tree channel Klein bottle amplitude allows to determine the normalization of the corresponding crosscap states

$$|C\rangle = 2^{(d/2-4)} \left(\prod_{j=1}^{d} \sqrt{\frac{R_1^{(j)}}{R_2^{(j)}}} \right) \left(|C_{\rm NS}\rangle + |C_{\rm R}\rangle \right). \qquad (2.18)$$

2.4. Annulus amplitude

Next we calculate all contributions of open strings stretching between the various D$(9-d)$-branes, generically located at nontrivial relative angles. We will both include the

case, where the relative angle is vanishing, i.e. the background gauge flux is equal on both branes, and the case, where the angle is $\pi/2$ and the field gets infinitely large on, say, p of the tori.

We start with the contributions of strings with both ends on the same brane. The T-dual of the Kaluza-Klein and winding spectrum in eq.(2.7) reads

$$M_\mu^2 = \sum_{j=1}^{d} \left(\left(\frac{r_\mu^{(j)}}{V_\mu^{(j)}} \right)^2 + \left(s_\mu^{(j)} \right)^2 \left(\frac{R_1^{(j)} R_2^{(j)}}{V_\mu^{(j)}} \right)^2 \right) \tag{2.19}$$

with

$$V_\mu^{(j)} = \sqrt{\left(R_1^{(j)} n_\mu^{(j)} \right)^2 + \left(R_2^{(j)} m_\mu^{(j)} \right)^2} \tag{2.20}$$

denoting the volume of the brane on $T^2_{(j)}$. It is now straightforward to compute the loop channel annulus amplitude for open strings starting and ending on the same brane and transform it to the tree channel

$$\widetilde{A}_{\mu\mu} = c\, N_\mu^2 (1-1) \int_0^\infty dl\, \frac{1}{2^{(d+1)}} \prod_{j=1}^{d} \frac{\left(V_\mu^{(j)} \right)^2}{R_1^{(j)} R_2^{(j)}}\, \frac{\vartheta\left[\begin{smallmatrix} 1/2 \\ 0 \end{smallmatrix}\right]^4}{\eta^{12}} \sum_{r,s} e^{-\pi l \widetilde{M}_\mu^2} \tag{2.21}$$

with

$$\widetilde{M}_\mu^2 = \sum_{j=1}^{d} \left(\left(r_\mu^{(j)} \right)^2 \left(V_\mu^{(j)} \right)^2 + \left(s_\mu^{(j)} \right)^2 \left(\frac{V_\mu^{(j)}}{R_1^{(j)} R_2^{(j)}} \right)^2 \right). \tag{2.22}$$

N_μ counts the numbers of different kinds of branes. Using (2.21) one can determine the normalization of the boundary state, which has the schematic form

$$|D_\mu\rangle = 2^{-(d/2+1)} \left(\prod_{j=1}^{d} \frac{\left(V_\mu^{(j)} \right)^2}{R_1^{(j)} R_2^{(j)}} \right) \left(|D_{\mu,\mathrm{NS}}\rangle + |D_{\mu,\mathrm{R}}\rangle \right). \tag{2.23}$$

Reflecting the brane on a single $T^2_{(j)}$ by a π rotation onto itself corresponds to $(n_\mu^{(j)}, m_\mu^{(j)}) \rightarrow (-n_\mu^{(j)}, -m_\mu^{(j)})$ and, as can be determined in the boundary state approach, changes the sign of the RR charge, thus exchanging branes and anti-branes. Using the boundary state (2.23) we can compute the tree channel annulus amplitude for an open string stretched between two different D-branes

$$\widetilde{A}_{\mu\nu} = \int_0^\infty dl\, \langle D_\mu | e^{-lH_{cl}} | D_\nu \rangle =$$

$$\frac{1}{2} c\, N_\mu N_\nu I_{\mu\nu} \int_0^\infty dl\, (-1)^d \sum_{\alpha,\beta \in \{0,1/2\}} (-1)^{2(\alpha+\beta)} \frac{\vartheta\left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix}\right]^{4-d} \prod_{j=1}^{d} \vartheta\left[\begin{smallmatrix} \alpha \\ (\phi_\nu^{(j)} - \phi_\mu^{(j)})/\pi + \beta \end{smallmatrix}\right]}{\eta^{12-3d} \prod_{j=1}^{d} \vartheta\left[\begin{smallmatrix} 1/2 \\ (\phi_\nu^{(j)} - \phi_\mu^{(j)})/\pi + 1/2 \end{smallmatrix}\right]}, \tag{2.24}$$

where the coefficient

$$I_{\mu\nu} = \prod_{j=1}^{d} \left(n_\mu^{(j)} m_\nu^{(j)} - m_\mu^{(j)} n_\nu^{(j)} \right) \tag{2.25}$$

is the (oriented) intersection number of the two branes. It gives rise to an extra multiplicity in the annulus loop channel, which we have to take into account, when we compute the massless spectrum. In order to properly include the case where some $\phi_\mu^{(j)} = \phi_\nu^{(j)}$, one needs to employ the relation

$$\lim_{\psi \to 0} \frac{2\sin(\pi\psi)}{\vartheta\left[{1/2 \atop 1/2+\psi}\right]} = -\frac{1}{\eta^3} \tag{2.26}$$

and include a sum over KK momenta and windings as in (2.21). The contribution to the massless RR tadpole due to (2.21) and (2.24) is

$$\int_0^\infty dl\, N_\mu N_\nu\, 2^{(3-d)} \prod_{j=1}^{d} \left(\frac{\left(R_1^{(j)}\right)^2 n_\mu^{(j)} n_\nu^{(j)} + \left(R_2^{(j)}\right)^2 m_\mu^{(j)} m_\nu^{(j)}}{R_1^{(j)} R_2^{(j)}} \right). \tag{2.27}$$

The loop channel annulus can be obtained by a modular transformation

$$\begin{aligned} A_{\mu\nu} = c\, N_\mu N_\nu\, I_{\mu\nu} \int_0^\infty \frac{dt}{t^{(6-d)}} \frac{1}{4} \sum_{\alpha,\beta \in \{0,1/2\}} (-1)^{2(\alpha+\beta)} e^{2i\alpha \sum_j (\phi_\nu^{(j)} - \phi_\mu^{(j)})} e^{i\pi d/2} \\ \times \frac{\vartheta\left[{-\beta \atop \alpha}\right]^{4-d} \prod_{j=1}^{d} \vartheta\left[{-(\phi_\nu^{(j)} - \phi_\mu^{(j)})/\pi - \beta \atop \alpha}\right]}{\eta^{12-3d} \prod_{j=1}^{d} \vartheta\left[{-(\phi_\nu^{(j)} - \phi_\mu^{(j)})/\pi - 1/2 \atop 1/2}\right]}. \end{aligned} \tag{2.28}$$

Of course, one can alternatively start from the loop channel, putting in the intersection numbers as an extra multiplicity by hand. The loop channel annulus amplitude looks like a twisted open string sector and considering for instance the NS sector, $\alpha = \beta = 0$, of (2.28) one can expand the ϑ-functions in (2.28) as

$$\frac{\vartheta\left[{0 \atop 0}\right]^{4-d}}{\eta^{12-d}} \prod_{j=1}^{d} \left[\left(\sum_{l^{(j)} \geq 1} q^{\epsilon^{(j)} l^{(j)}} \right) \frac{\prod_{n^{(j)} \geq 1} (1 + q^{n^{(j)} - \epsilon^{(j)} - \frac{1}{2}})(1 + q^{n^{(j)} + \epsilon^{(j)} - \frac{1}{2}})}{\prod_{n^{(j)} \geq 1} (1 - q^{n^{(j)} - \epsilon^{(j)}})(1 - q^{n^{(j)} + \epsilon^{(j)}})} \right], \tag{2.29}$$

with $\epsilon^{(j)} = (\phi_\mu^{(j)} - \phi_\nu^{(j)})/\pi$. The non-negative integers $l^{(j)}$ correspond to the Landau-levels in [23].

2.5. Möbius amplitude

Computing the overlap between the crosscap state (2.18) and a boundary state (2.23) yields the contribution of the brane $D(9-p)_\mu$ to the Möbius amplitude

$$\begin{aligned}\widetilde{M}_\mu = \mp c\, N_\mu\, 2^5\, (-1)^d \int_0^\infty dl \prod_{j=1}^{d} m_\mu^{(j)} \\ \times \sum_{\alpha,\beta\in\{0,1/2\}} (-1)^{2(\alpha+\beta)} \frac{\vartheta\big[{\alpha \atop \beta}\big]^{4-d} \prod_{j=1}^{d} \vartheta\big[{\alpha \atop \phi_\mu^{(j)}/\pi+\beta}\big]}{\eta^{12-3d} \prod_{j=1}^{d} \vartheta\big[{1/2 \atop \phi_\mu^{(j)}/\pi+1/2}\big]},\end{aligned} \tag{2.30}$$

with argument $q=-\exp(-4\pi l)$. Therefore the contribution to the RR tadpole is

$$\mp \int_0^\infty dl\, N_\mu\, 2^{(9-d)} \prod_{j=1}^{d} \left(\frac{R_1^{(j)}}{R_2^{(j)}} n_\mu^{(j)} \right). \tag{2.31}$$

The overall sign in (2.30) and (2.31) is fixed by the tadpole cancellation condition. In the loop channel the contribution of the Möbius strip results from strings starting on one brane and ending on its mirror partner. The extra multiplicity given by the numbers $m_\mu^{(j)}$ of intersection points invariant under $\mathcal{R}$ needs to be regarded as before. Now we have all the ingredients to study the relations which derive from the cancellation of massless RR tadpoles.

3. Compactifications to six dimensions

We are compactifying type I strings on a four-dimensional torus and cancel the tadpoles by introducing stacks of D9-branes with magnetic fluxes. The T-dual arrangement of D7-branes at angles looks like the situation depicted in figure 2, where we have drawn only two types of D7-branes labelled by μ and ν and their mirror partners μ' and ν', the angles being chosen arbitrary.

3.1. Six-dimensional models

The complete annulus amplitude is a sum over all open strings stretched between the various D7-branes

$$\begin{aligned}\widetilde{A}_{tot} = \sum_{\mu=1}^{K} \left(\widetilde{A}_{\mu\mu} + \widetilde{A}_{\mu'\mu'} + \widetilde{A}_{\mu\mu'} + \widetilde{A}_{\mu'\mu} \right) + \\ \sum_{\mu<\nu} \left(\widetilde{A}_{\mu\nu} + \widetilde{A}_{\nu\mu} + \widetilde{A}_{\mu'\nu'} + \widetilde{A}_{\nu'\mu'} + \widetilde{A}_{\mu\nu'} + \widetilde{A}_{\nu\mu'} + \widetilde{A}_{\mu'\nu} + \widetilde{A}_{\nu'\mu} \right).\end{aligned} \tag{3.1}$$

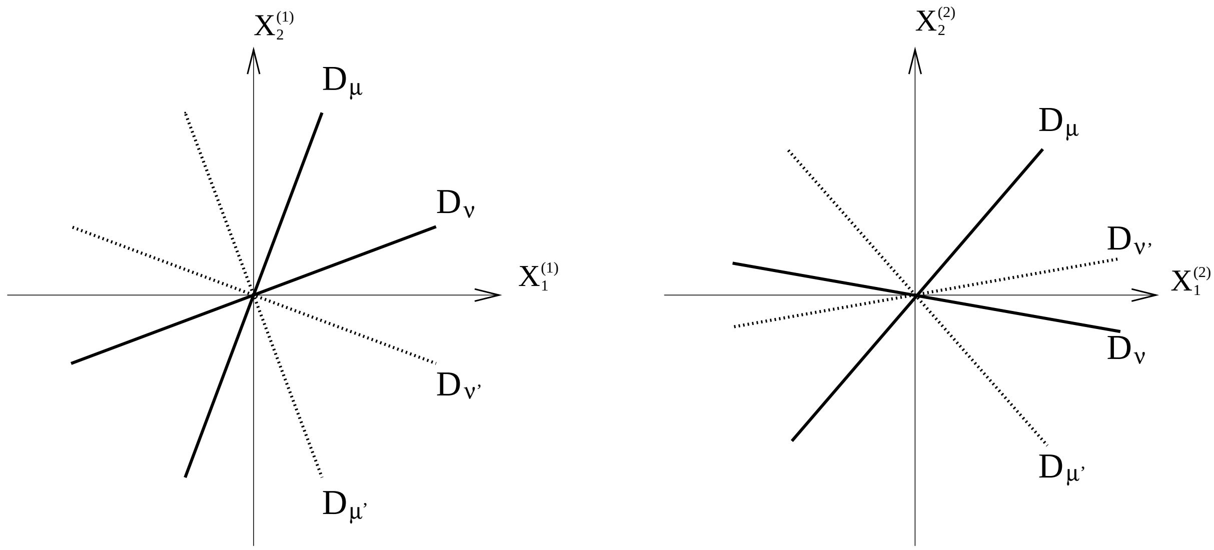

Figure 2

Using (2.21) and (2.27) and adding up all these various contributions yields the following two RR tadpoles

$$\int_0^\infty dl\, 8\left(\frac{R_1^{(1)}R_1^{(2)}}{R_2^{(1)}R_2^{(2)}}\right)\left(\sum_{\mu=1}^K N_\mu\, n_\mu^{(1)} n_\mu^{(2)}\right)^2 + \int_0^\infty dl\, 8\left(\frac{R_2^{(1)}R_2^{(2)}}{R_1^{(1)}R_1^{(2)}}\right)\left(\sum_{\mu=1}^K N_\mu\, m_\mu^{(1)} m_\mu^{(2)}\right)^2 . \tag{3.2}$$

For the total Möbius amplitude we obtain the RR tadpole

$$\mp\int_0^\infty dl\, 2^8\, \frac{R_1^{(1)}R_1^{(2)}}{R_2^{(1)}R_2^{(2)}} \sum_{\mu=1}^K N_\mu\, n_\mu^{(1)} n_\mu^{(2)}. \tag{3.3}$$

Note, the two special cases of N_9 horizontal and N_5 vertical D7-branes are contained in (3.2) and (3.3) by setting $N_\mu = N_9/2$ respectively $N_\mu = N_5/2$. Choosing the minus sign in (3.3) we get the two RR tadpole cancellation conditions

$$\begin{aligned} \frac{R_1^{(1)}R_1^{(2)}}{R_2^{(1)}R_2^{(2)}}: &\qquad \sum_{\mu=1}^K N_\mu\, n_\mu^{(1)} n_\mu^{(2)} = 16, \\ \frac{R_2^{(1)}R_2^{(2)}}{R_1^{(1)}R_1^{(2)}}: &\qquad \sum_{\mu=1}^K N_\mu\, m_\mu^{(1)} m_\mu^{(2)} = 0. \end{aligned} \tag{3.4}$$

As one might have expected, pure D9-branes with $m_\mu^{(j)} = 0$ only contribute to the tadpole proportional to the volume of the torus, while the D5-branes with $n_\mu^{(j)} = 0$ to the one proportional to the inverse volume. Remarkably, by choosing multiple winding numbers,

$n_\mu^{(j)} > 1$, one can reduce the rank of the gauge group. As usual in non-supersymmetric models, there remains an uncancelled NSNS tadpole, which needs to be cancelled by a Fischler-Susskind mechanism.

In the next section we shall show that except for the trivial case, when $m_\mu^{(1)} = m_\mu^{(2)} = 0$ for all μ, i.e. vanishing gauge flux on all the D9-branes, supersymmetry is broken and tachyons develop for open strings stretched between different branes. In contrast to the breaking of supersymmetry in a brane-antibrane system these tachyons cannot be removed by turning on Wilson-lines, which is related via T-duality to shifting the position of the branes by some constant vector. At any non trivial angle there always remains an intersection point of two D7-branes where the tachyons can localize. Also the lowest lying bosonic spectrum depends on the radii of the torus, which determine the relative angles. The zero point energy in the NS sector of a string stretching between two different branes is shifted by

$$\Delta E_{0,\mathrm{NS}} = \frac{1}{2} \sum_{j=1}^{d} \frac{\phi_\mu^{(j)} - \phi_\nu^{(j)}}{\pi} \tag{3.5}$$

using the convention $\phi_\mu^{(j)} - \phi_\nu^{(j)} \in (0, \pi/2]$. Even assuming a standard GSO projection, the lightest physical state can easily be seen to be tachyonic except for the supersymmetric situation with $\phi_\mu^{(1)} - \phi_\nu^{(1)} = \phi_\mu^{(2)} - \phi_\nu^{(2)}$. We shall find in the next section that tadpole cancellation prohibits this solution, except when all fluxes vanish.

On the contrary, the chiral fermionic massless spectrum is independent of the moduli and we display it in table 1.

spin	rep.	number
$(1,2)$	$\mathbf{A}_\mu + \overline{\mathbf{A}}_\mu$	$4m_\mu^{(1)} m_\mu^{(2)}$
$(1,2)$	$\mathbf{A}_\mu + \overline{\mathbf{A}}_\mu + \mathbf{S}_\mu + \overline{\mathbf{S}}_\mu$	$2m_\mu^{(1)} m_\mu^{(2)} (n_\mu^{(1)} n_\mu^{(2)} - 1)$
$(1,2)$	$(\mathbf{N}_\mu, \mathbf{N}_\nu) + (\overline{\mathbf{N}}_\mu, \overline{\mathbf{N}}_\nu)$	$(n_\mu^{(1)} m_\nu^{(1)} + m_\mu^{(1)} n_\nu^{(1)})(n_\mu^{(2)} m_\nu^{(2)} + m_\mu^{(2)} n_\nu^{(2)})$
$(1,2)$	$(\mathbf{N}_\mu, \overline{\mathbf{N}}_\nu) + (\overline{\mathbf{N}}_\mu, \mathbf{N}_\nu)$	$(n_\mu^{(1)} m_\nu^{(1)} - m_\mu^{(1)} n_\nu^{(1)})(n_\mu^{(2)} m_\nu^{(2)} - m_\mu^{(2)} n_\nu^{(2)})$

Table 1: *Chiral 6D massless open string spectrum.*

($\mathbf{A}_\mu$ and $\mathbf{S}_\mu$ denote the antisymmetric resp. symmetric tensor representations with respect to $U(N_\mu)$, $SO(N_\mu)$ or $Sp(N_\mu)$.) Since $\Omega\mathcal{R}$ exchanges a brane with its mirror brane, the Chan-Paton indices of strings ending on a stack of branes with non-vanishing gauge flux

have no Ω projection and the gauge group is $U(N_\mu)$. If $\Omega\mathcal{R}$ leaves branes invariant, i.e. the flux vanishes or is infinite, corresponding to pure D9- or D5-branes, the gauge factor is $SO(N_\mu)$ or $Sp(N_\mu)$, respectively.

The degeneracy of states stated in the third column of table 1 is essentially given by the intersection numbers of the D7-branes. Whenever it is formally negative, one has to pick the $(2,1)$ spinor of opposite chirality taking into account the opposite orientation of the branes at the intersection. As was pointed out earlier, a change of the orientation switches the RR charge in the tree channel translating into the opposite GSO projection in the loop channel. Therefore the other chirality survives the GSO projection in the R sector. If the multiplicity is zero, this does not mean that there are no massless open string states in this sector, it only means that the spectrum is not chiral. This happens precisely when two branes lie on top of each other in one of the two $T^2_{(j)}$ tori. Then the extra zero modes give rise to an extra spinor state of opposite chirality. The chiral spectrum shown in table 1 does indeed cancel the irreducible R^4 and F^4 anomalies.

We have also considered a $\mathbb{Z}_2$ orbifold background, together with nonvanishing magnetic flux, which changes the transversality condition to

$$\frac{R_2^{(1)} R_2^{(2)}}{R_1^{(1)} R_1^{(2)}} : \qquad \sum_{\mu=1}^{K} N_\mu \, m_\mu^{(1)} m_\mu^{(2)} = 16 \tag{3.6}$$

and leads to a projection $SO(N_\mu),\ Sp(N_\mu) \to U(N_\mu/2)$ on pure D9- and D5-branes but no further changes on D9-branes with nonvanishing flux. In this background it appears to be possible to construct also supersymmetric models [28].

3.2. Supersymmetry Breaking

One might suspect that there exist nontrivial configurations of D-branes cancelling all tadpoles while preserving maybe a reduced number of supersymmetries. Now, we would like to show that no such nontrivial supersymmetric configurations exist. We assume the absence of anti-branes from the beginning as their presence breaks supersymmetry anyway.

Going to the T-dual picture with D-branes at angles, we can apply the results of [29] for the supersymmetry preserved by two D-branes intersecting at some relative angles $\phi^{(1)}$ and $\phi^{(2)}$ on the two tori. Whenever the square of the operation Θ which rotates one brane onto the other has eigenvalues 1 when acting on the supercharges

$$\Theta^2 Q_\alpha = Q_\alpha, \tag{3.7}$$

some supersymmetry is preserved. Here Q_α denote the sixteen tendimensional spinor states with internal quantum numbers $(\pm 1/2, \pm 1/2, ...)$ and some definite chirality. Hence, Θ^2 has eigenvalues $\exp(\pm i\phi^{(1)} \pm i\phi^{(2)})$ being equal to 1 exactly if $\phi^{(1)} = \pm\phi^{(2)} \bmod 2\pi$, which preserves the supercharges $\pm(1/2, -1/2, ...)$ or $\pm(1/2, 1/2, ...)$ respectively. An important point to notice is, that the angles must not be measured modulo π, but instead the relative orientations of the respective branes need to be regarded and a consistent convention in measuring angles only modulo 2π in $(-\pi, \pi]$ must be adopted. For this purpose, on each $T^2_{(j)}$ we associate the direction of the vector $(n^{(j)}_\mu R^{(j)}_1, m^{(j)}_\mu R^{(j)}_2)$ to the respective D7-brane. To preserve supersymmetry, one of the two conditions, $\phi^{(1)} = \pm\phi^{(2)} \bmod 2\pi$, must hold in any open string sector. Therefore all branes need to be at equal or opposite angles on the two tori.

Now let us first consider brane configurations without D5-branes, i.e. without vertical D7-branes, and concentrate on the second condition in (3.4), which we call the transversality condition. We are free to choose $m^{(1)}_\mu \geq 0$. In order to have any chance to satisfy the transversality condition, there is at least one pair of branes, $D7_1$ and $D7_2$, with $m^{(2)}_1 \geq 0$ and $m^{(2)}_2 \leq 0$. One can easily convince oneself, that whenever $D7_1$ and $D7_2$ have relative angles $\phi^{(1)} = \pm\phi^{(2)}$ this cannot be the true for either $D7_1$ and $D7_{2'}$ or $D7_{1'}$ and $D7_2$.

We still need to include vertical D7-branes to complete the proof. They have a positive contribution to the transversality condition in (3.4). In order to get a net negative contribution from D7-branes at angles in $(0, \pi/2)$ relative to the $X^{(j)}_1$ axes, we need $\pm m^{(1)}_\mu \geq 0$ and $\mp m^{(2)}_\mu \geq 0$ for some μ. These branes have $\phi^{(1)} = -\phi^{(2)}$ relative to their mirrors, preserving $\pm(1/2, 1/2, ...)$, but at best may only have $\phi^{(1)} = \pi - \phi^{(2)}$ relative to the vertical D7-branes. Thus, such configurations are not supersymmetric, either.

Such a situation has been illustrated in figure 3. If one flips the orientation of the vertical D7-branes on the second torus, the configuration turns supersymmetric, as all brane sectors now satisfy $\phi^{(1)} = -\phi^{(2)}$. But one finds, that in order to cancel the non-abelian anomaly then, one is required to take a different chirality for the D5-D9 strings as compared to the rest of the spectrum and therefore the sector behaves as in the presence of an anti-D5-brane. This refers to the change of the GSO projection in the loop channel induced by flipping the orientation.

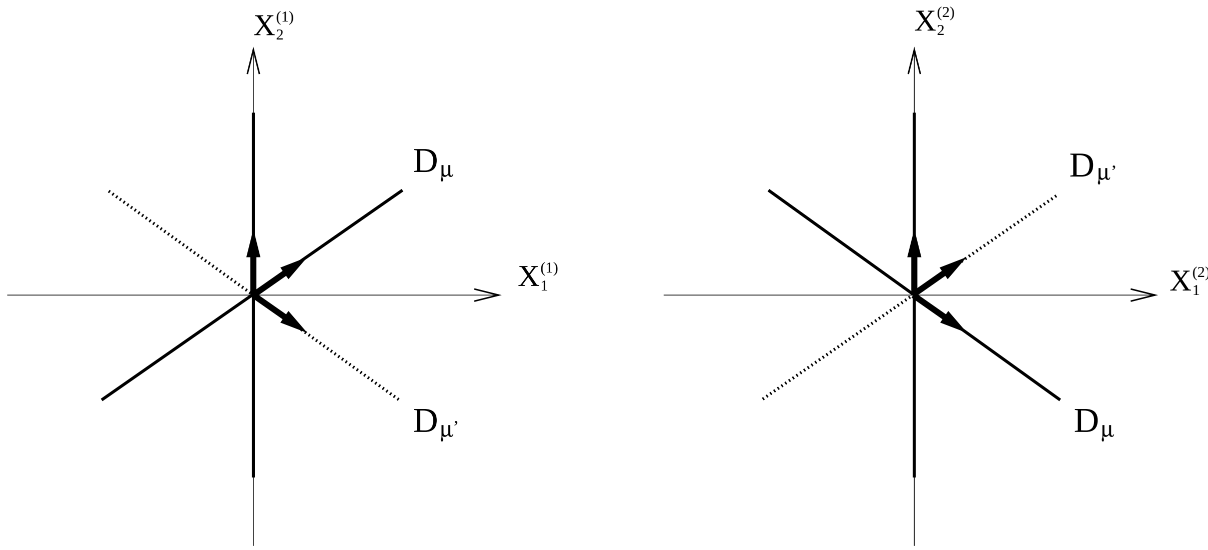

Figure 3

4. Four dimensional models

The completely analogous computation as in six dimensions can be performed for the compactification of type I strings on a 6-torus in the presence of additional gauge fields. Now we cancel the tadpoles by D9-branes with magnetic fluxes on all three 2-tori respectively, in the T-dual picture, by D6-branes at angles. One obtains four independent tadpole cancellation conditions

$$\begin{aligned}
\frac{R_1^{(1)}R_1^{(2)}R_1^{(3)}}{R_2^{(1)}R_2^{(2)}R_2^{(3)}}: &\qquad \sum_{\mu=1}^{K} N_\mu\, n_\mu^{(1)} n_\mu^{(2)} n_\mu^{(3)} = 16 \\
\frac{R_1^{(1)}R_2^{(2)}R_2^{(3)}}{R_2^{(1)}R_1^{(2)}R_1^{(3)}}: &\qquad \sum_{\mu=1}^{K} N_\mu\, n_\mu^{(1)} m_\mu^{(2)} m_\mu^{(3)} = 0 \\
\frac{R_2^{(1)}R_1^{(2)}R_2^{(3)}}{R_1^{(1)}R_2^{(2)}R_1^{(3)}}: &\qquad \sum_{\mu=1}^{K} N_\mu\, m_\mu^{(1)} n_\mu^{(2)} m_\mu^{(3)} = 0 \\
\frac{R_2^{(1)}R_2^{(2)}R_1^{(3)}}{R_1^{(1)}R_1^{(2)}R_2^{(3)}}: &\qquad \sum_{\mu=1}^{K} N_\mu\, m_\mu^{(1)} m_\mu^{(2)} n_\mu^{(3)} = 0.
\end{aligned} \tag{4.1}$$

(For convenience they are given in the picture with D6-branes at angles.) Again the gauge group contains a $U(N_\mu)$ factor for each stack of D9-branes with non-vanishing flux, an $SO(N_\mu)$ gauge factor for a stack with vanishing flux and an $Sp(N_\mu)$ factor for a stack of D5-branes. The general spectrum of chiral fermions with respect to the gauge group factors is presented in table 2.

rep.	number
$(\mathbf{A}_\mu)_L$	$8m_\mu^{(1)}m_\mu^{(2)}m_\mu^{(3)}$
$(\mathbf{A}_\mu)_L+(\mathbf{S}_\mu)_L$	$4m_\mu^{(1)}m_\mu^{(2)}m_\mu^{(3)}(n_\mu^{(1)}n_\mu^{(2)}n_\mu^{(3)}-1)$
$(\mathbf{N}_\mu,\mathbf{N}_\nu)_L$	$(n_\mu^{(1)}m_\nu^{(1)}+m_\mu^{(1)}n_\nu^{(1)})(n_\mu^{(2)}m_\nu^{(2)}+m_\mu^{(2)}n_\nu^{(2)})(n_\mu^{(3)}m_\nu^{(3)}+m_\mu^{(3)}n_\nu^{(3)})$
$(\overline{\mathbf{N}}_\mu,\mathbf{N}_\nu)_L$	$(n_\mu^{(1)}m_\nu^{(1)}-m_\mu^{(1)}n_\nu^{(1)})(n_\mu^{(2)}m_\nu^{(2)}-m_\mu^{(2)}n_\nu^{(2)})(n_\mu^{(3)}m_\nu^{(3)}-m_\mu^{(3)}n_\nu^{(3)})$

Table 2: *Left-handed 4D massless open string spectrum.*

Whenever the intersection number in the second column is formally negative, one again has to take the conjugate representation. The spectrum in table 2 is free of non-abelian gauge anomalies. The remarks concerning the bosonic NS part of the spectrum made above for six dimensions also apply here. Masses depend on the radii and we have not been able to produce an otherwise consistent model free of tachyons or even preserving supersymmetry.

In the next subsections we discuss some examples and point out some phenomenological issues for these models.

4.1. A 24 generation SU(5) model

Having found a way to break supersymmetry, to reduce the rank of the gauge group and to produce chiral spectra in four space-time dimensions, it is tempting to search in a compact bottom-up approach for brane configurations producing massless spectra close to the standard model. The tachyons are not that dangerous from the effective field theory point of view, as they simply may serve as Higgs-bosons for spontaneous gauge symmetry breaking, anticipating a mechanism to generate a suitable potential keeping their vacuum expectation values finite. In [23] a three generation GUT model was presented, which we shall revisit in the following. The gauge group of the model is $G=U(5)\times U(3)\times U(4)\times U(4)$ with maximal rank, so that we have to choose all $n_\mu^{(j)}=1$. The following choice of $m_\mu^{(j)}$ then satisfies all tadpole cancellation conditions (4.1):

$$m_\mu^{(j)}=\begin{pmatrix}3 & -5 & 1 & -1\\ 1 & 1 & -1 & -1\\ 1 & 1 & 1 & 1\end{pmatrix}. \tag{4.2}$$

This configuration of D6-branes is displayed in figure 4, where the mirror branes have been omitted. The chiral part of the fermionic massless spectrum is shown in table 3.

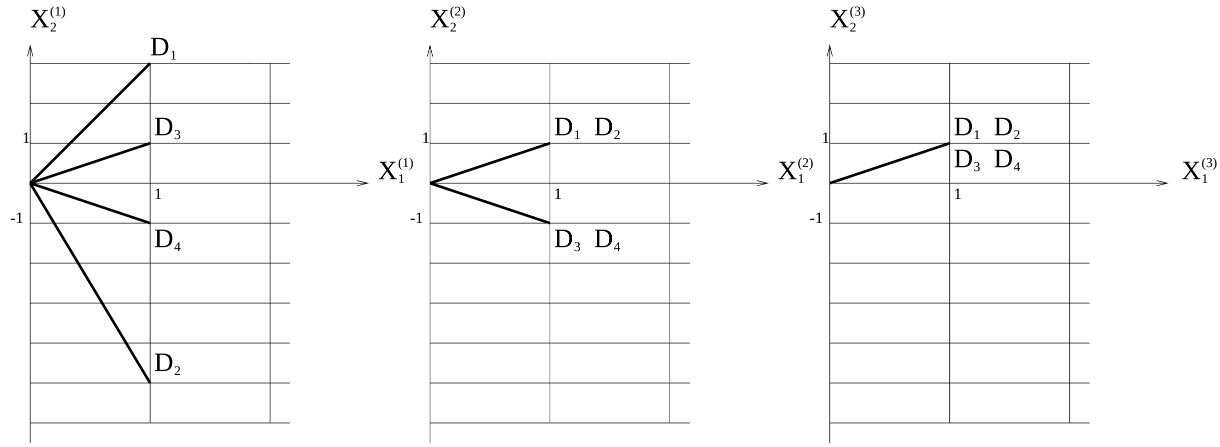

Figure 4

rep.	number
$(\mathbf{10},\mathbf{1},\mathbf{1},\mathbf{1})$	24
$(\mathbf{1},\mathbf{3},\mathbf{1},\mathbf{1})$	40
$(\overline{\mathbf{5}},\overline{\mathbf{3}},\mathbf{1},\mathbf{1})$	8
$(\mathbf{1},\mathbf{1},\overline{\mathbf{6}},\mathbf{1})$	8
$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{6})$	8

Table 3: *Chiral left-handed fermions for the 24 generation model.*

No chiral fermions transform under both the $U(5)\times U(3)$ gauge group and the $U(4)\times U(4)$ gauge group, but there will of course also be non-chiral bifundamentals. If we think of the $SU(5)$ factor as a GUT gauge group, then this model has 24 generations[4]. We shall see in the following that it is actually impossible to get a model with three or any odd number of generations in this framework.

4.2. A four generation model

The tadpole cancellation condition

$$\sum_{\mu=1}^{K} N_\mu\, n_\mu^{(1)} n_\mu^{(2)} n_\mu^{(3)} = 16 \tag{4.3}$$

[4] In [23] this model was advocated as a three generation model. We can formally reproduce the model in [23] by dividing the matrix (4.2) by a factor of two. However, this is inconsistent as it would violate the condition that the $m_\mu^{(j)}$'s have to be integers. Thus, we conclude that in string theory only the choice (4.2) is correct and the model is actually a 24 generation model.

tells us that we can reduce the rank of the gauge group right from the beginning by choosing some $n_\mu^{(j)} > 1$. Therefore, we can envision a model where we start with the gauge group $U(3) \times U(2) \times U(1)^r$ at the string scale. In order to have three quark generations in the $(\mathbf{3}, \mathbf{2})$ representation of $SU(3) \times SU(2)$, we necessarily need $I_{12} = 3$ and $I_{12'} = 0$. However, this is not possible, as $I_{\mu\nu} - I_{\mu\nu'}$ is always an even integer. The model we found closest to the 4 generation standard model is presented in the following. We choose the gauge group $U(3) \times U(2) \times U(1)^2$ and the following configuration of four stacks of D-branes:

$$n_\mu^{(j)} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 10 \end{pmatrix}, \qquad m_\mu^{(j)} = \begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 1 & -2 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}. \tag{4.4}$$

The configuration has been illustrated in figure 5.

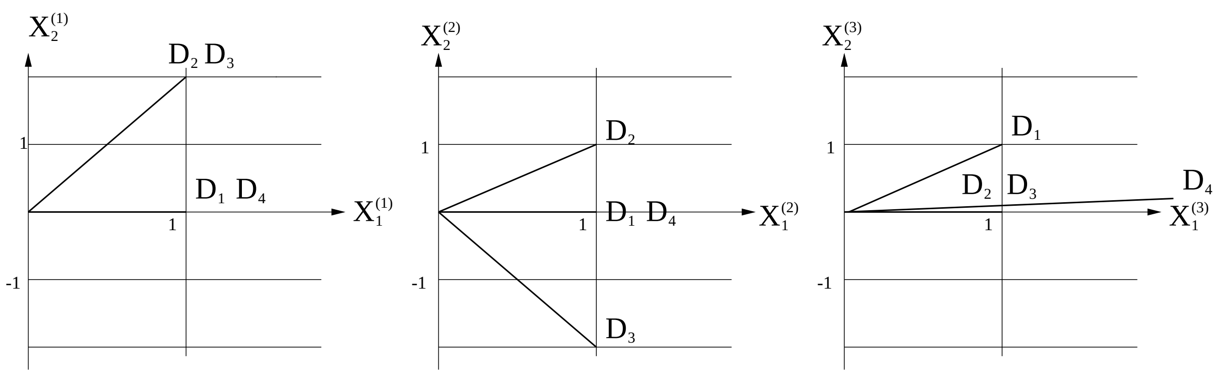

Figure 5

The resulting chiral massless spectrum is shown in table 4.

$SU(3) \times SU(2) \times U(1)^4$	number
$(\mathbf{3}, \mathbf{2})_{(1,1,0,0)}$	2
$(\mathbf{3}, \mathbf{2})_{(1,-1,0,0)}$	2
$(\overline{\mathbf{3}}, \mathbf{1})_{(-1,0,-1,0)}$	4
$(\overline{\mathbf{3}}, \mathbf{1})_{(-1,0,1,0)}$	4
$(\mathbf{1}, \mathbf{2})_{(0,1,0,1)}$	2
$(\mathbf{1}, \mathbf{2})_{(0,-1,0,1)}$	2
$(\mathbf{1}, \mathbf{1})_{(0,0,-1,-1)}$	4
$(\mathbf{1}, \mathbf{1})_{(0,0,1,-1)}$	4

Table 4: *Chiral left-handed fermions for the 4 generation model.*

Computing the mixed $G^2 - U(1)$ anomalies one realizes that one of the abelian gauge factors is anomalous, which needs to be cured by the Green-Schwarz mechanism. The other three anomaly-free abelian gauge groups include a suitable hypercharge $U(1)$

$$U(1)_Y = \frac{1}{3}U(1)_1 + U(1)_3 - U(1)_4, \tag{4.5}$$

so that the spectrum finally looks like the one in table 5.

$SU(3)\times SU(2)\times U(1)_Y\times U(1)^2$	number
$(\mathbf{3},\mathbf{2})_{(\frac{1}{3},1,0)}$	2
$(\mathbf{3},\mathbf{2})_{(\frac{1}{3},-1,0)}$	2
$(\overline{\mathbf{3}},\mathbf{1})_{(-\frac{4}{3},0,-1)}$	4
$(\overline{\mathbf{3}},\mathbf{1})_{(\frac{2}{3},0,1)}$	4
$(\mathbf{1},\mathbf{2})_{(-1,1,0)}$	2
$(\mathbf{1},\mathbf{2})_{(-1,-1,0,)}$	2
$(\mathbf{1},\mathbf{1})_{(0,0,-1)}$	4
$(\mathbf{1},\mathbf{1})_{(2,0,1)}$	4

Table 5: *Chiral left-handed fermions for the 4 generation model.*

We found a semi-realistic, non-supersymmetric, four generation standard-model like spectrum with two gauged flavour symmetries and right-handed neutrinos. In order to determine the Higgs sector, we would have to investigate the bosonic part of the spectrum. However, this is not universal but depends on the radii of the six-dimensional torus. We will not elaborate this further but instead discuss another important issue concerning the possible phenomenological relevance of these models.

Since we break supersymmetry already at the string scale M_s, in order to solve the gauge hierarchy problem we must choose M_s in the TeV region. Let us employ the T-dual picture of D6-branes at angles again to analyze the situation in more detail. Using the relations

$$M_{pl}^2 \sim \frac{M_s^8 V_6}{g_s^2}, \quad \frac{1}{(g_{\rm YM}^{(\mu)})^2} \sim \frac{M_s^3 V_\mu}{g_s}, \tag{4.6}$$

where V_μ denotes the volume of some D6-brane in the internal directions

$$V_\mu = \prod_{j=1}^{3} V_\mu^{(j)} \tag{4.7}$$

and $g_{\rm YM}^{(\mu)}$ the gauge coupling on this brane. They imply

$$M_s \sim \alpha_{\rm YM}^{(\mu)} M_{pl} \frac{V_\mu}{\sqrt{V_6}}, \tag{4.8}$$

Therefore, for the TeV scenario to work one needs

$$\frac{V_\mu}{\sqrt{V_6}} << 1 \tag{4.9}$$

for all D6-branes. However, chirality for the fermionic spectrum of an open string stretched between any two D6-branes implied that the two branes in question do not lie on top of each other on any of the three $T^2_{(j)}$ tori. In other words the two branes already span the entire torus and the condition (4.9) cannot be realized.

5. Conclusions

In this paper we have investigated type I string compactifications on noncommutative tori, which are due to constant magnetic fields along the world volumes of the D9-branes being wrapped around the internal space. The key features of these models in six and four dimensions are the presence of chiral fermions, low rank gauge groups and broken space-time supersymmetry. In fact, we found a four-dimensional model with standard model gauge group $SU(3)\times SU(2)\times U(1)_Y$ (times some Abelian flavour gauge groups) with four generations of standard model fermions and also a four-dimensional 24 generation $SU(5)$ GUT-like model. On the other hand, while displaying promising features like getting chiral spectra, these simple models suffer from other phenomenological difficulties, like even numbers of generations and the problem of splitting the string and the Planck scale sufficiently. It remains to be seen whether more complicated backgrounds with magnetic flux can improve this situation.

Since the deformation parameters of the noncommutative tori, we have considered, correspond to rational (up to some trivial volume dependence) magnetic fluxes, it was always possible to choose a T-dual description where the internal space is commutative, but instead the various D-branes intersect at particular rational angles. It would be interesting to find also noncommutative compactifications which do not allow for an equivalent, T-dual commutative description. It would be also interesting to discuss the properties of the effective quantum field theories which originate from noncommutative compactifications along the lines of ref.[30].

Acknowledgements

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References

[1] E. Witten, *Search for realistic Kaluza-Klein theory*, Nucl. Phys. **B186** (1981) 412.

[2] P. Candelas, G. Horowitz, A. Strominger and E. Witten, *Vacuum configurations for superstrings*, Nucl. Phys. **B258** (1985) 46.

[3] L. Dixon, J.A. Harvey, C. Vafa and E. Witten, *Strings on orbifolds*, Nucl. Phys. **B261** (1985) 678.

[4] W. Lerche, D. Lüst and A.N. Schellekens, *Chiral, four-dimensional heterotic strings from self-dual lattices*, Nucl. Phys. **B287** (1987) 477.

[5] H. Kawai, D. Lewellen and S.H. Tye, *Construction of fermionic string models in four dimensions*, Nucl. Phys. **B288** (1987) 1; I. Antoniadis, C. Bachas and C. Kounnas, *Four-dimensional superstrings*, Nucl. Phys. **B289** (1987) 87.

[6] M. Douglas and G. Moore, *D-branes, quivers and ALE instantons*, hep-th 9603167; I.R. Klebanov and E. Witten, *Superconformal field theory on three-branes at a Calabi-Yau singularity*, Nucl. Phys. **B536** (1998) 199, hep-th/9807080.

[7] K. Landsteiner, E. Lopez and D. Lowe, *Duality of chiral $N = 1$ supersymmetric gauge theories via branes*, JHEP **9802** (1998) 007, hep-th/9801002; I. Brunner, A. Hanany, A. Karch and D. Lüst, *Brane dynamics and chiral non-chiral transitions*, Nucl. Phys. **B528** (1998) 197, hep-th/9801017; A. Hanany and A. Zaffaroni, *On the realization of chiral four-dimensional gauge theories using branes*, JHEP **9805** (1998) 001, hep-th/9801134.

[8] C. Angelantonj, M. Bianchi, G. Pradisi, A. Sagnotti and Y. Stanev, *Chiral asymmetry in four-dimensional open string vacua*, Phys. Lett. **B385** (1996) 96, hep-th/9606169.

[9] P. Horava and E. Witten, *Heterotic and type I string dynamics from eleven dimensions*, Nucl. Phys. **B460** (1996) 506, hep-th/9510209; E. Witten, *Strong coupling expansion of Calabi-Yau compactification*, Nucl. Phys. **B471** (1996) 135, hep-th/9602070; M. Faux, D. Lüst and B.A. Ovrut, *Intersecting orbifold planes and local anomaly cancellation in M-theory*, Nucl. Phys. **B554** (1999) 437, hep-th/9903028; V. Kaplunovsky, J. Sonnenschein, S. Theisen and S. Yankielowicz, *On the duality between perturbative orbifolds and M-theory on T^4/Z_N*, hep-th/9912144; M. Faux, D. Lüst and B.A. Ovrut, *Local Anomaly Cancellation, M-theory orbifolds and phase transitions*, hep-th/0005251.

[10] J.P. Derendinger, L.E. Ibanez and H.P. Nilles, *On the low-energy $D = 4$, $N = 1$ supergravity theory extracted from the $D = 10$, $N = 1$ superstring*, Phys. Lett. **B155** (1985) 65; M. Dine, R. Rohm, N. Seiberg and E. Witten, *Gluino condensation in superstring models*, Phys. Lett. **B156** (1985) 55.

[11] J. Scherk, J.H. Schwarz, *Spontaneous breaking of supersymmetry through dimensional reduction*, Phys. Lett. **B82** (1979) 60; *How to get masses from extra dimensions*, Nucl. Phys. **B153** (1979) 61; E. Cremmer, J. Scherk, J.H. Schwarz, *Spontaneously broken*

$\mathcal{N} = 8$ *supergravity*, Phys. Lett. **B84** (1979) 83; S. Ferrara, C. Kounnas, M. Porrati, F. Zwirner, *Superstrings with spontaneously broken supersymmetry and their effective theories*, Nucl. Phys. **B318** (1989) 75.

[12] I. Antoniadis, E. Dudas, A. Sagnotti, *Supersymmetry breaking, open strings and M-theory*, Nucl. Phys. **B544** (1999) 469, hep-th/9807011; I. Antoniadis, G. D'Appollonio, E. Dudas, A. Sagnotti, *Partial breaking of supersymmetry, open strings and M-theory*, Nucl. Phys. **B553** (1999) 133, hep-th/9812118; *Open descendants of $\mathbb{Z}_2 \times \mathbb{Z}_2$ freely acting orbifolds*, Nucl. Phys. **B565** (2000) 123, hep-th/9907184.

[13] C. Angelantonj, *Non-tachyonic open descendants of the 0B string theory*, Phys. Lett. **B444** (1998) 309, hep-th/9810214; R. Blumenhagen, A. Font and D. Lüst, *Tachyon free orientifolds of type 0B strings in various dimensions*, Nucl. Phys. **B558** (1999) 159, hep-th/9904069; R. Blumenhagen, A. Font, A. Kumar and D. Lüst, *Aspects of type 0 string theory*, Class. Quant. Grav. **17** (2000) 989, hep-th/9908155; R. Blumenhagen and A. Kumar, *A Note on Orientifolds and Dualities of Type 0B String Theory*, Phys.Lett. **B464** (1999) 46, hep-th/9906234.

[14] I. Antoniadis, E. Dudas and A. Sagnotti, *Brane supersymmetry breaking*, Phys. Lett. **B464** (1999) 38, hep-th/9908023; G. Aldazabal and A.M. Uranga, *Tachyon-free Non-supersymmetric Type IIB Orientifolds via Brane-Antibrane Systems*, JHEP **9910** (1999) 024, hep-th/9908072; G. Aldazabal, L.E. Ibanez and F. Quevedo, *Standard-like Models with Broken Supersymmetry from Type I String Vacua*, JHEP **0001** (2000) 031, hep-th/9909172; *A D-Brane Alternative to the MSSM* , hep-ph/0001083; C. Angelantonj, I. Antoniadis, G. D'Appollonio, E. Dudas, A. Sagnotti, *Type I vacua with brane supersymmetry breaking*, hep-th/9911081; C. Angelantonj, R. Blumenhagen and M.R. Gaberdiel, *Asymmetric orientifolds, brane supersymmetry breaking and non BPS branes*, hep-th/0006033.

[15] E. Witten; *New issues in manifolds of $SU(3)$ holonomy*, Nucl. Phys. **B268** (1986) 79.

[16] L.E. Ibanez, H.P. Nilles and F. Quevedo, *Reducing the rank of the gauge group in orbifold compactifications of the heterotic string*, Phys. Lett. **B192** (1987) 332.

[17] A. Connes, M.R. Douglas and A. Schwarz, *Noncommutative Geometry and Matrix Theory: Compactification on Tori*, JHEP **9802** (1998) 003, hep-th/9711162; M.R. Douglas and C. Hull, *D-branes and the Noncommutative Torus*, JHEP **9802** (1998) 008, hep-th/9711165.

[18] R. Blumenhagen, L. Görlich, B. Körs and D. Lüst, *Asymmetric Orbifolds, Noncommutative Geometry and Type I Vacua*, hep-th/0003024.

[19] C.G. Callan, C. Lovelace, C.R. Nappi and S.A. Yost, *String Loop Corrections To Beta Functions*, Nucl. Phys. **B288** (1987) 525; A. Abouelsaood, C.G. Callan, C.R. Nappi and S.A. Yost, *Open Strings in Background Gauge Fields*, Nucl. Phys. **B280** (1987) 599; Y.-K.E. Cheung and M. Krogh, *Noncommutative Geometry from D0-branes in a Background B-field*, Nucl.Phys. **B528** (1998) 185, hep-th/9803031; J. Fröhlich, O.

Grandjean and A. Recknagel, *Supersymmetric Quantum Theory and Noncommutative Geometry*, Commun. Math. Phys. **203** (1999) 119, math-ph/9807006; F. Ardalan, H. Arfaei and M. M. Sheikh-Jabbari, *Dirac Quantization of Open Strings and Noncommutativity in Branes*, hep-th/9906161; C.-S. Chu and P.-M. Ho, *Noncommutative Open String and D-brane*, Nucl.Phys. **B550** (1999) 151, hep-th/9812219; *Constrained Quantization of Open String in Background B Field and Noncommutative D-brane*, hep-th/9906192; J. Madore, S. Schraml, P. Schupp and J. Wess, *Gauge theory on noncommutative spaces*, hep-th/0001203; B. Jurco, P. Schupp and J. Wess, *Noncommutative gauge theory for Poisson manifolds*, hep-th/0005005.

[20] V. Schomerus, *D-branes and Deformation Quantization*, JHEP **9906** (1999) 030, hep-th/9903205.

[21] N. Seiberg and E. Witten, *String Theory and Noncommutative Geometry*, JHEP **9909** (1999) 032, hep-th/9908142.

[22] F. Ardalan, H. Arfaei and M. M. Sheikh-Jabbari, *Noncommutative Geometry From Strings and Branes*, JHEP **9902** (1999) 016, hep-th/9810072.

[23] C. Bachas, *A way to break supersymmetry*, hep-th/9503030.

[24] R. Blumenhagen, L. Görlich and B. Körs, *Supersymmetric Orientifolds in 6D with D-Branes at Angles*, Nucl. Phys. **B569** (2000) 209, hep-th/9908130; *A New Class of Supersymmetric Orientifolds with D-Branes at Angles*, hep-th/0002146; *Supersymmetric 4D Orientifolds of Type IIA with D6-branes at Angles*, JHEP **0001** (2000) 040, hep-th/9912204.

[25] I. Antoniadis, E. Kiritsis and T.N. Tomaras, *A D-brane alternative to unification*, hep-th/0004214; G. Aldazabal, L. E. Ibanez, F. Quevedo and A. M. Uranga, *D-Branes at Singularities : A Bottom-Up Approach to the String Embedding of the Standard Model*, hep-th/0005067; A. Krause, *A small cosmological constant, grand unification and warped geometry*, hep-th/0006226.

[26] B. Chen, H. Itoyama, T. Matsuo and K. Murakami, *p-p' System with B-field, Branes at Angles and Noncommutative Geometry*, `hep-th/9910263`.

[27] C.-S. Chu, *Noncommutative Open String: Neutral and Charged*, hep-th/0001144.

[28] C. Angelantonj, I. Antoniadis, E. Dudas and A. Sagnotti, in preparation.

[29] M. Berkooz, M.R. Douglas and R.G. Leigh, *Branes Intersecting at Angles*, Nucl. Phys. **B480** (1996) 265, hep-th/9606139.

[30] J. Gomis, T. Mehen and M.B. Wise, *Quantum field theories with compact noncommutative extra dimensions*, hep-th/0006160.